Proceedings of the 26th Annual International Conference of the IEEE EMBS
San Francisco, CA, USA • September 1-5, 2004

# Power Spectral Density Estimation and Tracking of Nonstationary Pressure Signals based on Kalman Filtering

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***Abstract*— We describe an algorithm to estimate and track slow changes in power spectral density (PSD) of nonstationary pressure signals. The algorithm is based on a Kalman filter that adaptively generates an estimate of the autoregressive model parameters at each time instant. The algorithm exhibits superior PSD tracking performance in nonstationary pressure signals than classical nonparametric methodologies, and does not assume a piecewise stationary model of the data. Furthermore, it provides better time–frequency resolution, and is robust to model mismatches. We demonstrate its usefulness by a sample application involving PSD estimation and tracking of short records of simulated pressure waveforms. This algorithm is intended for applications were the PSD must be estimated and tacked during short transient periods, possibly after clinical interventions.**

***Keywords*— Kalman Filter, spectral estimation, linear models, arterial blood pressure, intracranial pressure.**

## I. Introduction

CURRENTLY, power spectral density (PSD) estimation of physiologic signals is performed predominantly using classical techniques based on the Fast Fourier Transform (FFT). Nonparametric methods such as the periodogram and its improvements (i.e. Barlett's, Welch's, and Blackman–Tukey's methodologies [1]–[4]) are based on the idea of estimating the autocorrelation sequence of a random process from measured data, and then taking the FFT to obtain an estimate of the power spectrum. The main two advantages of these techniques are their computational efficiency, due to the numerical efficiency of the FFT algorithm, and that they do not make any assumptions about the process except for its stationarity. However, these techniques have some limitations. They require stationarity of the segments studied, do not work well for short data records, and have limited frequency resolution. Since physiologic signals are nonstationary in nature, these techniques are applied following the methodology of the Short–Time Fourier Transform (STFT), where nonparametric methods are applied to short overlapping segments which are assumed to be stationary. This approach has also its limitations. It imposes a piecewise stationary model on the data and, and since local stationarity requires the analysis segments to be short in duration, they have limited time–frequency resolution.

Time–frequency resolution can be improved by using parametric methods of PSD estimation. The parametric approach is based on modeling the signal under analysis as a realization of a particular stochastic process and estimating the model parameters from its samples. Even though parametric PSD can improve the frequency resolution, the current techniques for PSD estimation based on AR models (i.e. autocorrelation, covariance, modified convariance, and Burg's methods [5]) assume stationarity. To analyze nonstationary signals they must also assume the signal is locally piecewise stationary. There are some applications were this may also be inadequate, for instance, in the analysis of transient responses to clinical interventions were the response is too short and time varying to assume local stationarity.

We describe a methodology to estimate the time-varying autoregressive (AR) model parameters of nonstationary pressure signals using an adaptive Kalman filter. This methodology produces instantaneous estimates of PSD, improved time–frequency resolution, and enables for nonstationary PSD tracking in situations where data records are too short and the local stationary model does not work well.

## II. Methods

The adaptive Kalman filter algorithm we propose for instantanous PSD estimation and tracking assumes an underlying autoregressive structure of the data. We chose an underlying AR model structure because of its intrinsic generality and peak matching capabilities. These are important properties for the analysis of physiologic signals, since we are usually more interested in estimating the frequency at which the formant frequencies (peaks) occur than the valleys. Starting from this assumption, we modeled a given physiologic pressure signal with a recursion of the form

$$x(n) = \sum_{k=1}^{P} a_k x(n-k) + w(n) \qquad (1)$$

where $x(n)$ is the pressure signal under analysis at instant $n$, $\{a_k\}_{k=1}^{p}$ are the model parameters, $\{x(n-k)\}_{k=1}^{p}$ are

This work was supported in part by the Thrasher Research Fund.

delayed samples of the signal, and $w(n)$ is assumed to be a random sequence independent and normally distributed with zero mean. Equation (1) can be generalized by allowing the model coefficients $\{a_k\}_{k=1}^{p}$ to be time–variant. The estimation problem within the context of nonstationary processes yields naturally to the discrete Kalman filter (DKF) [7]–[9].

In order to use the DKF we must have a signal model in state–space form, and the state of the system evolves as a first–order difference equation, and must be estimated from noisy observations. The general form of the state–space model for the linear DKF is given by [8], [10]

$$\begin{aligned} \mathbf{x}(n) &= \mathbf{A}(n-1)\mathbf{x}(n-1) + \mathbf{w}(n) \\ \mathbf{y}(n) &= \mathbf{H}(n)\mathbf{x}(n) + \mathbf{v}(n) \end{aligned} \quad (2)$$

where $\mathbf{x}(n)$ is the state of the system, $\mathbf{A}(n-1)$ is the transition or system matrix, $\mathbf{H}(n)$ is the observation matrix, $\mathbf{y}(n)$ is the vector of observations, and $\mathbf{w}(n)$ and $\mathbf{v}(n)$ are zero–mean white Gaussian noise processes representing system and observation noise, respectively. The system noise and the process noise are assumed to be independent. If the problem can be formulated in state–space according to (2), and if we know $\mathbf{A}(n-1)$, $\mathbf{H}(n)$, and the covariance matrix of $\mathbf{w}(n)$ and $\mathbf{v}(n)$, then we can use the DKF to estimate the state of the system optimally according to the Kalman recursion.

### A. Signal Model in State–Space

Since our signal model (1) is a $p^{th}$–order difference equation, we can transform it to a system of difference equations by defining the state of the system as a $p$–dimensional vector,

$$\mathbf{x}(n) = \begin{pmatrix} x(n) \\ x(n-1) \\ \vdots \\ x(n-p+1) \end{pmatrix}. \quad (3)$$

This enables us to rewrite (1) as a first–order difference equation with time–varying model parameters, and enables us to create a state–space model for the DKF,

$$\mathbf{A}(n) = \begin{pmatrix} a_1(n) & a_2(n) & \dots & a_p(n) \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \quad (4)$$

$$\mathbf{x}(n) = \mathbf{A}(n-1)\mathbf{x}(n-1) + \mathbf{w}(n) \quad (5)$$

$$y(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{v}(n). \quad (6)$$

The measurement matrix is $\mathbf{H} = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix}$. However, in order for this state-space model to be useful we need a way to estimate the vector of time-varying coefficients $\mathbf{a}(n) = \begin{pmatrix} a_1(n) & a_2(n) & \dots & a_p(n) \end{pmatrix}^T$ corresponding to the first row of the transition matrix at time $n$, $\mathbf{A}(n)$.

### B. Dual Kalman filter

The vector of time–varying coefficients that made the first row of the transition matrix $\mathbf{a}(n)$ can also be estimated using a DKF. This is referred as a dual Kalman filter, that is, two DKFs working in parallel to estimate the model parameters and the state of the system [11]–[14].

The estimation of the model parameters $\mathbf{a}(n)$ can be formulated in state space as follows,

$$\begin{aligned} \mathbf{a}(n) &= \mathbf{\Phi}\mathbf{a}(n-1) + \mathbf{e}(n) \\ x(n+1) &= \mathbf{x}^T(n)\mathbf{a}(n) + \mathbf{q}(n). \end{aligned} \quad (7)$$

where $\mathbf{\Phi}$ is a user specified diagonal matrix with entries $(\rho_{ij})_{i=j}$ corresponding to the correlation between $\mathbf{a}(n)$ and $\mathbf{a}(n-1)$, which control the adaptation speed and frequency tracking capabilities of the algorithm. For biomedical signals, where the model parameters change slowly, values close to 1 work well (i.e. 0.995). In the case of $(\rho_{ij})_{i=j} = 1$, the system equation becomes $\mathbf{a}(n) = \mathbf{a}(n-1) + \mathbf{e}(n)$. This is a simple Markov process where the vector coefficients evolve following a random walk. The adaptation speed is controlled by the covariance of $\mathbf{e}(n)$. There is a tradeoff between high adaptation speed (fast tracking) and variance of the estimates. The measurement equation of the model, $x(n+1) = \mathbf{x}^T(n)\mathbf{a}(n) + \mathbf{q}(n)$ implements a linear predictor, where the signal at time $n+1$ is estimated from previous values of $n$ according to

$$\begin{aligned} x(n+1) &= \mathbf{x}^T(n)\mathbf{a}(n) + \mathbf{q}(n) = \\ &= \sum_{k=1}^{p} a_k(n)x(n-k+1) + \mathbf{q}(n). \end{aligned} \quad (8)$$

In the state–space formulation given by (7), the model parameters $\mathbf{a}(n)$ become state variables. The optimum linear estimate of state of the system $\mathbf{a}(n)$ can be estimated recursively according to,

$$\hat{\mathbf{a}}(n|n-1) = \mathbf{\Phi}\hat{\mathbf{a}}(n-1|n-1) \quad (9)$$

$$z(n) = x(n+1) \quad (10)$$

$$\hat{z}(n) = \mathbf{x}^T(n)\hat{\mathbf{a}}(n|n-1) \quad (11)$$

$$\hat{\mathbf{a}}(n|n) = \hat{\mathbf{a}}(n|n-1) + \mathbf{K}(n)\Big[z(n) - \hat{z}(n)\Big] \quad (12)$$

$$\mathbf{P}(n|n-1) = \mathbf{\Phi}\mathbf{P}(n-1|n-1)\mathbf{\Phi}^T + \mathbf{Q}_e(n) \quad (13)$$

$$\mathbf{\Xi}(n) = \mathbf{x}(n)^T\mathbf{P}(n|n-1)\mathbf{x}(n)^T \quad (14)$$

$$\mathbf{K}(n) = \mathbf{P}(n|n-1)\mathbf{x}(n)\Big[\mathbf{\Xi}(n) + \mathbf{Q}_l(n)\Big]^{-1} \quad (15)$$

$$\mathbf{P}(n|n) = \Big[\mathbf{I} - \mathbf{K}(n)\mathbf{x}^T(n)]\mathbf{P}(n|n-1)\Big] \quad (16)$$

where $\hat{\mathbf{a}}(n|n-1) = \mathbf{\Phi}\hat{\mathbf{a}}(n-1|n-1)$ is the best estimate of the state (i.e. AR model parameters) without incorporating the observation at time $n$, just based on the model structure we imposed on the evolution of $\mathbf{a}(n)$ (prediction), and $\hat{z}(n) = \mathbf{x}^T(n)\hat{\mathbf{a}}(n|n-1)$ is the best estimate of the the measurement $z(n) = x(n+1)$ based on the model. The optimum estimate of the state at time $n$ incorporating the measurement at time $n$ is given by $\hat{\mathbf{a}}(n|n) = \hat{\mathbf{a}}(n|n-1) + \mathbf{K}(n)\Big[z(n) - \hat{z}(n)\Big]$, which is composed of two terms: the best estimate of the state without the measurement at time $n$, and a weighted

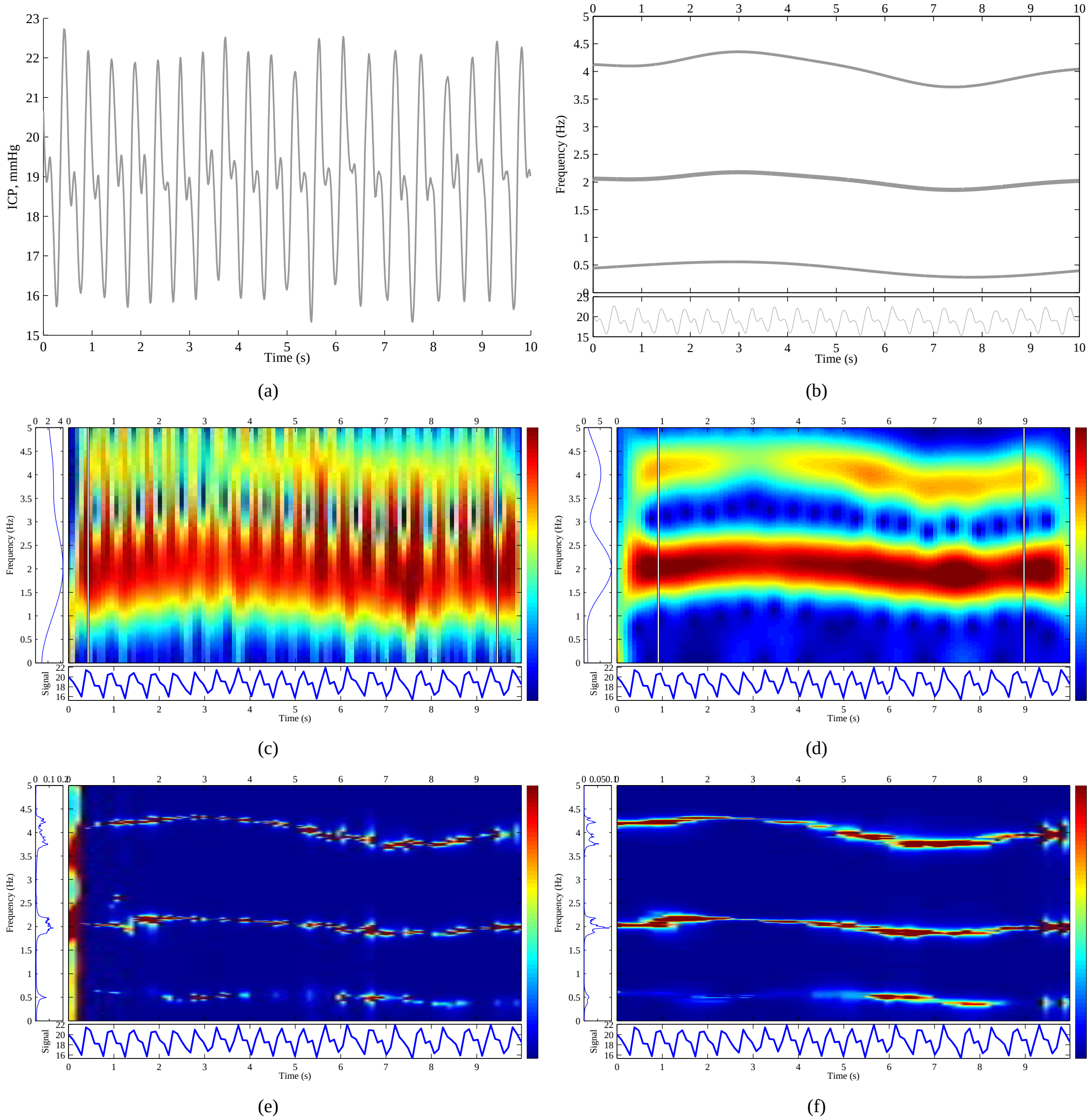


Fig. 1. Representative results of the comparative study between the nonparametric PSD estimator based on the STFT and the proposed DKF PSD estimator. (a) Realization of the nonstationary pressure signal (10 s). (b) Theoretical time-frequency content. Note the respiratory, fundamental cardiac component, and second harmonic of the cardiac component. (c) Nonparametric spectrogram using a 1 s. window (125 samples). (b) Nonparametric spectrogram using a 2 s window (250 samples). (e) DKF PSD estimate and tracking. (f) Smooth DKF PSD estimate and tracking using a 0.08 s window (10 samples)

difference of the observation at time $n$ and the best estimate of this observation (correction). The weighting factor $\mathbf{K}(n)$ is calculated optimally following the Kalman recursion, and is referred to as the Kalman gain [7], [8], [10].

### C. Instantanous PSD Estimation and Tacking

Since the dual Kalman filter we propose provides estimates of $\{a\}_{k=1}^{p}$ at each time instant, the nonstationary power spectrum is given by

$$\hat{P}_x(e^{jw}, n) = \frac{|\hat{b}(0,n)|^2}{|1 + \sum_{k=1}^{p} \hat{a}_k(n) e^{-jkw}|^2} \tag{17}$$

Therefore, the nonstationary PSD given the instantaneous estimates of model parameters $\mathbf{a}(n)$ can be computed according to,

$$\hat{P}_x(e^{jw}, n)_{KM} = \frac{1}{|\mathrm{FFT}\left[\bar{\mathbf{a}}(\mathbf{n})\right]|^2} \tag{18}$$

$$\bar{\mathbf{a}}(\mathbf{n}) = \begin{pmatrix} 1 & -\mathbf{a}(n)^T \end{pmatrix} \tag{19}$$

These estimates can be averaged over short windows (e.g. 10 samples) to improve the statistical properties of the DKF PSD estimator.

## III. Results and Discussion

We tested the reliability of the instantanous PSD estimation algorithm with synthetic data generated from different models (AR, MA, ARMA, and harmonic), and with real data from physiologic pressure signals. In the following, we demonstrate its usefulness by a sample application involving PSD estimation and tracking of short nonstationary simulated pressure signals.

Synthetic pressure signals were generated using the model in (20). The model incorporates the effects of pulse pressure variation as a conventional amplitude modulation (AM) of a multi-frequency pulse pressure carrier with respiration as the modulating signal, and an additive effect,

$$p(t) = u_p + \left[1 + a r_n(t)\right] \cdot \sum_{n=1}^{N} C_n e^{j2\pi f_c n t} + k r(t) \qquad (20)$$

where $r_n(t)$ is the normalized respiratory signal, $a$ is the modulation index, and the carrier signal is a quasi-periodic signal with an arbitrary pulse morphology that can be approximated as a multi-harmonic periodic signal with a fundamental frequency of $f_c$, corresponding to the cardiac frequency. The respiratory signal $r_n(t)$ was modeled as a multi-harmonic signal with a fundamental frequency equal to the respiratory rate $f_r$. The cardiac and respiratory frequencies, $f_c$ and $f_r$, were modeled as a sum of two components: a constant carrier frequency $\bar{f}$ an a stochastic frequency variation $\lambda(t)$,

$$\begin{aligned} f_r(t) &= \bar{f}_r + \lambda_r(t), \lambda_r(t) = -\sum_{k=1}^{P_1} a_k \sigma_r(t-k) + w(t) \\ f_c(t) &= \bar{f}_c + \lambda_c(t), \lambda_c(t) = -\sum_{k=1}^{P2} b_k \lambda_c(t-k) + w(t) \\ &+ \sum_{k=0}^{Q} h_c(k) \lambda_r(t-k) \qquad (21) \end{aligned}$$

where the cardiac $\lambda_c(t)$ and the respiratory $\lambda_r(t)$ stochastic frequency variations were modeled as two correlated autoregressive (AP) processes. Finally, the pressure signal is passed through a fading multipath channel to incorporate the nonstationary pulse pressure variability typical of real ICP and ABP signals

We compared PSD estimates obtained with the proposed Kalman PSD estimation algorithm with those generated by classical nonparametric estimation techniques. For the purposes of this study we used the modified periodogram method of nonparametric PSD estimation as the methodology representing the nonparametric methods.

The study was aimed at comparing the time–frequency resolution of the spectrograms generated using both methodologies on short records on nonstationary synthetic pressure signals. For this purpose, we generated an ensemble of short nonstationary pressure signals using the statistical model describe earlier.

In Fig. 1 we show representative results for a specific realization of the pressure signal. This comparative study between a nonparametric PSD estimator and the proposed PSD estimator based on the DKF was aimed at comparing their time–frequency resolution for nonstationary pressure signals. Fig. 1(a) shows a realization of the nonstationary pressure signal (10 s), and Fig. 1(b) shows its theoretical time-frequency content. Note the respiratory, fundamental cardiac component, and second harmonic of the cardiac component. Fig. 1(c) and Fig. 1(d) show the nonparametric spectrogram using a 1 s. window (125 samples) and using a 2 s window (250 samples), respectively. Fig. 1(e) and Fig. 1(f) show the instantaneous DKF PSD estimate and a smooth DKF PSD estimate using a 0.08 s window (10 samples).

Based on this preliminary study we conclude that the DKF estimator is able to tack PSD changes in pressure signals better than a moving window technique, and exhibits good time-frequency resolution when compared with nonparametric PSD techniques at the task of estimating the PSD of very short data records which are nonstationary. Furthermore, the proposed method does not assume a piecewise stationary model on the data, and can be used to study transient responses to events (e.g. clinical interventions) where the periods of analysis are too short but still nonstationary, and classical techniques cannot be used since the locally stationary assumption does not hold.

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